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TRANSIENT ANTIPLANE VIBRATIONS OF A RECTANGULAR ELASTIC SLAB*

N.V. BOYEV and M.A. SUMBATYAN

The non-stationary antiplane problem of an elastic rectangle under specified stresses on its lateral edge is considered. A solution of the problem in Laplace **transforms has been** obtained in the form of a series of homogeneous solutions. The use of certain special operator relationships enables one to write out the original of the solution in an explicit form. When this is done for any instant of time, each homogeneous solution is expressed in the form of a finite sum. A numerical analysis of the problem is presented and the characteristic features of the behaviour of the stressed state in time are established.

Let us consider the transient vibrations of an elastic slab of infinite length (the y-axis) and rectangular cross-section with sides of $2h$ and $2a$, $z\in[-h,h]$, $x\in[-a,a]$ under conditions of antiplane deformation caused by forces acting on opposite lateral edges $\xi=\pm1$, $z\in[-h,h]$

$$\tau_{\xi y}(\xi,\zeta,T)\,|_{\xi=\pm1}=f(\zeta,T);\ \xi=x/a,\ \zeta=z/h;\ \xi,\zeta\in[-1,1] \tag{1}$$

Here $f(\zeta,T)$ is an arbitrary function of the variables ζ and the time T. For simplicity, let us assume that the edges $\zeta=\pm1$ are free from stresses $(\tau_{\zeta y}|_{\zeta=\pm1}=0)$ and the initial conditions are:

$$v|_{T=0}=\partial v/\partial T\,|_{T=0}=0,\ 1<\xi,\ \zeta<1.$$

$(v=v(\xi,\zeta,T)$ is the displacement along the y-axis).

Let us apply a Laplace transform with respect to time to the initial boundary value problem. We have

$$\varepsilon^2\frac{\partial^2V}{\partial\xi^2}+\frac{\partial^2V}{\partial\zeta^2}=\varepsilon^2p^2V,\quad V=\int_0^\infty v(\xi,\zeta,t)\,e^{-st}\,dt$$
$$t=T/T_0,\ \varepsilon=h/a,\ p^2=\rho a^2s^2/(\mu T_0^2)$$

Here, ρ is the density of the material, μ is the shear modulus and T_0 is the characteristic time. Since, subject to condition (1), the function V is odd with respect to ξ, we shall seek it in the form $W(\zeta,p)\operatorname{sh}\gamma\xi$, where the function $W(\zeta,p)$ is determined from the following selfadjoint boundary value problem

$$\frac{d^2}{d\zeta^2}W(\zeta,p)=-\varepsilon^2(\gamma^2-p^2)W(\zeta,p),\quad \frac{d}{d\zeta}W(\pm1,p)=0 \tag{2}$$

In the case of a problem which is symmetric with respect to ζ, the eigenfunctions of problem (2) have the form

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$$W(\zeta, p) = B(p) \cos \varepsilon \sqrt{\gamma^2 - p^2}\,\zeta \tag{3}$$

where $B(p)$ is an unknown function of the parameter p and the eigenvalues are determined from the equation

$$\sqrt{\gamma^2 - p^2} \sin \varepsilon \sqrt{\gamma^2 - p^2} = 0, \quad \gamma_k^2 = \pi^2 k^2/\varepsilon^2 + p^2, \quad k = 0, \pm 1, \pm 2, \ldots$$

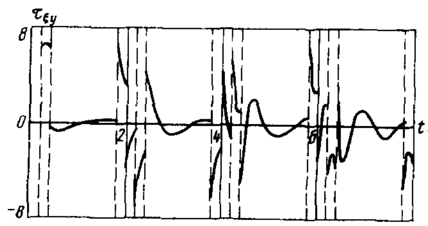

Fig.1

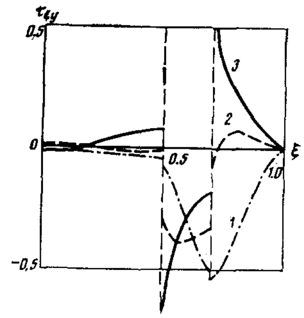

Fig.2

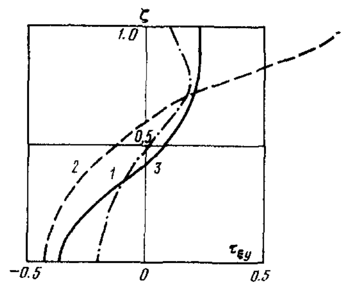

Fig.3

Hence, the solution of the initial boundary value problem is represented by a series of the homogeneous solutions

$$V(\xi, \zeta, p) = \sum_{k=0}^{\infty} B_k(p)\, W_k(\zeta; \gamma, p) \operatorname{sh} \gamma\xi$$

where the functions $W_k(\zeta; \gamma, p)$ have the form (3) (when $\gamma = \gamma_k$) and satisfy the orthogonality condition.

Let us determine the functions $B_k(p)$ which satisfy the boundary conditions on the edges $\xi = \pm 1$ using the orthogonality condition.

For example, in the case of a function $f(\zeta, t)$ which is even with respect to ζ, the Laplace transform has the form

$$\sigma_{\xi y}(\xi, \zeta, p) = \sum_{m=0}^{\infty} \delta_m \frac{\operatorname{ch} \gamma_m \xi}{\operatorname{ch} \gamma_m} \cos m\pi\zeta \int_{-1}^{1} F(\zeta_1, p) \cos m\pi\zeta_1 \, d\zeta_1 \tag{4}$$

$$\delta_0 = 1/2, \quad \delta_m = 1 \ (m \neq 0)$$

Here $F(\zeta, p)$ is the Laplace transform of the function $f(\zeta, t)$ with respect to the variable t.

Formula (4) is treated using the representation

$$\frac{F(\zeta, p)\, e^{-br}}{1 + e^{-2r}} = \sum_{n=0}^{\infty} (-1)^n F(\zeta, p)\, e^{-(b+2n)r}, \quad r = \sqrt{p^2 + a^2}$$

of the Laplace transform formula /1/

$$\underset{(b>0)}{e^{-bp} - e^{-br}} \doteq \begin{cases} 0, & 0 < t < b \\ abJ_1(ay)/y, & t > b, \quad y = \sqrt{t^2 - b^2} \end{cases}$$

and the convolution theorem.

Let us now give the expressions for the stresses

$$\tau_{\xi y}(\xi, \zeta, t) = \sum_{m=0}^{\infty} \delta_m \cos \frac{m\pi}{\varepsilon}\zeta \int_{-1}^{1} \cos \frac{m\pi}{\varepsilon}\zeta_1 \, d\zeta_1 \sum_{i=1}^{2} \sum_{0 \leqslant n < R_i} (-1)^n \times \tag{5}$$

$$\left[f(\zeta_1, R_i - 2n) - \frac{m\pi}{\varepsilon} P_{in} \int_0^t \frac{f(\zeta_1, t - \tau)}{(t^2 - P_{in}^2)^{1/2}} J_1\left(\frac{m\pi}{\varepsilon}(t^2 - P_{in}^2)^{1/2}\right) \times \begin{Bmatrix} 0, & 0 < t < P_{in} \\ 1, & t > P_{in} \end{Bmatrix} d\tau \right]$$

$$R_1 = 1/2\,(t - 1 + \xi), \quad R_2 = 1/2\,(t - 1 - \xi), \quad P_{1n} = 1 - \xi + 2n, \quad P_{2n} = 1 + \xi + 2n$$

In the case of series (5) which has been obtained, it is characteristic that each homogeneous solution is represented in the form of a sum for any instant of time t.

We now present the results of numerical calculations in the case when

$$\varepsilon = 1,\ f(\zeta, t) = \varphi(\zeta) g(t),\ \varphi(\zeta) = \zeta^2,\ g(t) = H(t) - H(t - 0.2)$$

where $H(t)$ is the Heaviside function. The time a/c is adopted for T_0 where $c = \sqrt{\mu/\rho}$ is the velocity of propagation of shear waves.

The stress distribution $\tau_{\xi y}$ in time at the point $\xi = \zeta = 0.8$ (Fig.1) reflects the transient nature of the change in the stressed state. In the interval $t \in [0.2, 0.4]$, the behaviour of the stresses $\tau_{\xi y}$ at this point is only slightly different from the behaviour of the function $g(t)$. However, during the interval when the perturbations are reflected from the boundaries $\xi = \pm 1$, the stresses change rapidly both in magnitude and in sign. During the intervals of time after the passage of the rear front of the wave as time increases, the oscillation of the stresses increases and changes sign.

The distribution of the stresses with respect to ξ at the instant of time $t = 2.5$ when $\zeta = 0, 0.5, 1$ (curves *1*, *2* and *3*) is shown in Fig.2. When $\zeta = 0$, if the "jumps" in the stresses when $\xi = 0.5$ and $\xi = 0.7$ are insignificant in magnitude, then, as ζ increases, they become larger and are accompanied by a change in sign.

When $t = 2.5$, curves *1-3* in Fig.3 reflect the stress distribution with respect to ζ when $\xi = 0.2$ (curve *1* $(10\tau_{\xi y})$, $\xi = 0.5$ (curve *2*, $(10\tau_{\xi y})$, and $\xi = 0.8$ (curve *3*).

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AN APPROACH TO SOLVING THE PROBLEM OF A CRACK IN A WEDGE-SHAPED PART OF A PLANE*

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In a development of previous obtained results of the solution of a problem concerning a crack which emerges orthogonally onto the boundary of a half-plane /1/, the problem of a crack of finite length on the axis of symmetry of one of the wedge-shaped parts of a plane is considered. The indices of the singularities of the solution are determined at both vertices of the crack and expressions are presented for the coefficients accompanying these singularities. Numerical values of the coefficients of the stress intensity are obtained in the case when the parts are opened at a right angle and there is a constant load on the edge of the crack. These results are in agreement with data cited in the literature for a piecewise homogeneous plane with a slit which emerges orthogonally onto the line where the half-planes join /2/.

Previously /3/, a solution of the functional Wiener-Hopf equation was presented in closed form for an analogous problem and an expression was given for the coefficient accompanying the fractional power singularity of the solution, that is, at the right end of the slit. Most attention will therefore be paid to isolating the singularities of the two vertices of the crack and to determining the coefficients accompanying these singularities.

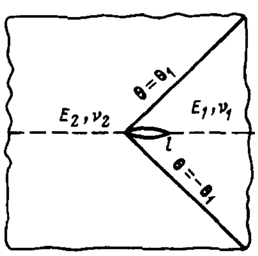

Fig.1

1. Formulation of the problem. Reduction to the Riemann problem.

A crack of finite length is considered which emerges along the axis of symmetry of one of the wedge-shaped parts of a piecewise homogeneous plane onto the line where the materials are joined (Fig. 1). A selfbalanced load $\sigma_\theta(r, 0) = -f(r)$ is applied to the edges of

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